#### SUPPLEMENTAL MATERIAL

Here we describe the setup of our experiments.

For all experiments, we use the SIREN network with the ADAM optimizer and no scheduler.

# Taylor-Green

Experiment.

Domain.

$$x \in [0, 2\pi], y \in [0, 2\pi]$$

Initial Velocity.

$$u(x, y) = (\sin(x)\cos(y), -\cos(x)\sin(y))$$

Parameters.

• learning rate:  $10^{-5}$ 

• layer size: 6

• number of layers: 64

• timestep ( $\Delta t$ ): 0.001

• max iterations: 10000

• batch size: 64<sup>2</sup>

• cutoff function  $\varepsilon$ : 0.01

• screening weight: 350

Comparison experiment. All previous methods INSR [Chen et al. 2023b], PINN [Raissi et al. 2019], and piDeepONet [Wang et al. 2021] are run using the same parameters.

Domain.

$$x\in [0,2\pi], y\in [0,2\pi]$$

Initial Velocity.

$$u(x, y) = (\sin(x)\cos(y), -\cos(x)\sin(y))$$

Parameters.

• layer size: 3

• number of layers: 256

• timestep ( $\Delta t$ ): 0.001

• max iterations: 20000

• cutoff function  $\varepsilon$ : 0.01

• screening weight: 350

#### Von Kármán vortex street

Domain.

$$x \in [-1.1, 1.9], y \in [-0.6, 0.6]$$

Initial Velocity.

$$u(x, y) = (0.5, 0.0)$$

Obstacle. Circular obstacle with radius 0.04 and center (-0.8, 0.0)

Parameters.

• learning rate:  $10^{-5}$ 

• layer size: 2

• number of layers: 128

• timestep ( $\Delta t$ ): 0.05

• max iterations: 10000

• batch size: 128<sup>2</sup>

• cutoff function  $\varepsilon$ : 0.015

• screening weight: 350

## **Smoke**

Rising smoke plume.

Domain.

$$x \in [-1.0, 1.0], y \in [-1.0, 1.0], z \in [-1.0, 1.0]$$

Initial Velocity. Randomly supplied values within a range

$$(x, y, z) \in ((-0.1, 0.1), (-0.1, 0.1), (0.1, 0.3))$$

Parameters.

• learning rate:  $10^{-5}$ 

• layer size: 5

• number of layers: 64

• timestep ( $\Delta t$ ): 0.05

• max iterations: 10000

• batch size: 128<sup>3</sup>

• cutoff function  $\varepsilon$ : 0.01

• screening weight: 350

Smoke plume with obstacle.

Domain.

$$x \in [-1.0, 1.0], y \in [-1.0, 1.0], z \in [-1.0, 1.0]$$

Initial Velocity.

$$u(x, y, z) = (0.0, 0.0, 1.0)$$

Obstacle. Spherical obstacle with radius 0.1 and center (0.0, 0.0, -0.3)

Parameters.

• learning rate:  $10^{-5}$ 

• layer size: 5

• number of layers: 64

• timestep ( $\Delta t$ ): 0.05

• max iterations: 10000

• batch size: 128<sup>3</sup>

• cutoff function  $\varepsilon$ : 0.01

• screening weight: 350

Smoke ring.

Domain.

$$x \in [-1.0, 1.0], y \in [-1.0, 1.0], z \in [-1.0, 1.0]$$

Initial Velocity.

$$\theta = a\cos\left(\frac{x - 0.2}{\sqrt{(x - 0.2)^2 + (y - 2)^2}}\right)$$

$$u_1(x, y, z) = (0, 0, 0.2(1 + 0.01\cos(8\cos(\theta)))$$

$$u_2(x, y, z) = (0, 0, -0.2(1 + 0.01\cos(8\cos(\theta)))$$

 $u_1$  and  $u_2$  are defined only within spheres. First smoke sphere has center (0.0,0.0,-0.21) with radius 0.2. The second smoke sphere has center (0.0,0.0,0.21) with radius 0.2.

### Parameters.

 $\bullet$  learning rate:  $10^{-5}$ 

• layer size: 5

number of layers: 64
timestep (Δt): 0.05
max iterations: 10000

batch size: 128<sup>3</sup>
cutoff function ε: 0.01
screening weight: 350